Math 124 Finding Tangent Lines

Here are four standard problems from our Math 124 course about finding tangent lines. The first is the most standard example. The second involves parametric equations (so you need to know $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$). The third and fourth involve finding the tangent at an unknown point on the curve such that the line also passes through another point. Try these out and see solutions on the following pages.

1. Find the equation for the tangent line to $y = \sqrt{2x+4} + 3xe^{2x}$ at $x = 0$.

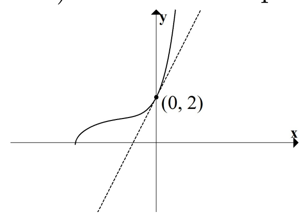
2. Find the equation of the tangent line to $x = 4t^2$, $y = 3t - t^3$ at the point where $t = 1$.

3. Find the equations of the two lines that are tangent to $y = x^2$ and also pass through (0,-6).

4. Find the equations of the two lines that are tangent to $x = 3t^2 + 2t$, $y = 5t^2 - t$ and also pass through (0, -4).

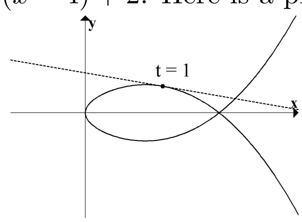
1. Find the equation for the tangent line to $y = \sqrt{2x+4} + 3xe^{2x}$ at $x = 0$.

   SOLUTION: At $x = 0$, we have $y(0) = \sqrt{2(0)+4} + 3(0)e^{2(0)} = 2$. Thus, the line goes through the point $(0, 2)$ and is of the form $y = m(x - 0) + 2$.
   Next, $\frac{dy}{dx} = \frac{2}{2\sqrt{2x+4}} + 3e^{2x} + 6xe^{2x}$. Thus, the slope at $x = 0$ is $y'(0) = \frac{1}{\sqrt{4}} + 3e^0 + 0 = \frac{7}{2}$.
   Therefore, the tangent line is $y = \frac{7}{2}(x - 0) + 2$. Here is a picture:



2. Find the equation of the tangent line to $x = 4t^2$, $y = 3t - t^5$ at the point where $t = 1$.

   SOLUTION: At $t = 1$, we have $x(1) = 4$ and $y(1) = 3 - 1 = 2$. Thus, the line goes through the point $(4, 2)$ and is of the form $y = m(x - 4) + 2$.
   Next, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3-5t^4}{8t}$. Thus, the slope at $t = 1$ is $\frac{3-5}{8} = -\frac{1}{4}$.
   Therefore, the tangent line is $y = -\frac{1}{4}(x - 4) + 2$. Here is a picture:



3. Find the equations of the two lines that are tangent to $y = x^2$ and also pass through (0,-6).

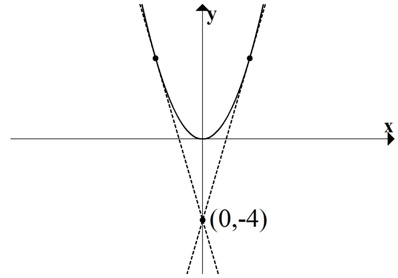
   SOLUTION: First label the unknown tangent points by (a, b). Now we write down all the conditions we are trying to satisfy:

   (a) (a, b) is on the curve. Thus, $b = a^2$.
   (b) The slope of the tangent is always $y' = 2x$, so at (a, b) the SLOPE OF TANGENT $= 2a$.
   (c) The desired line needs to go through (a, b) and $(0, -4)$, so DESIRED SLOPE $= \frac{b-(-6)}{a-0}$.

   We want the slope of the tangent at (a, b) to match the desired slope, so we want to solve

$$2a = \frac{b+6}{a} \qquad \text{and} \qquad b = a^2.$$

   Now we simplify, combine and solve as follows: $2a^2 = b + 6 \Rightarrow 2a^2 = a^2 + 6 \Rightarrow a^2 = 6 \Rightarrow a = \pm\sqrt{6}$.
   The corresponding slopes are $-2\sqrt{6}$ and $2\sqrt{6}$. Therefore the two tangent lines are $y = -2\sqrt{6}(x-0)+(-6)$ and $y = 2\sqrt{6}(x - 0) + (-6)$. Here is a picture:



4. Find the equations of the two lines that are tangent to $x = 3t^2 + 2t$, $y = -5t^2$ and also pass through (0,-4).

SOLUTION: Label the unknown tangent points by (a, b). Now we write down all the conditions we are trying to satisfy:

(a) (a, b) is on the curve. Thus, $a = 3t^2 + 2t$ and $b = -5t^2$.

(b) The SLOPE OF TANGENT $= \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-10t}{6t+2}$.

(c) The desired line needs to go through (a, b) and $(0, -4)$, so DESIRED SLOPE $= \frac{b-(-4)}{a-(0)}$.

We want the slope of the tangent at (a, b) to match the desired slope, so we solve

$$\frac{-10t}{6t+2} = \frac{b+4}{a} \qquad \text{and} \qquad a = 3t^2 + 2t, b = -5t^2.$$

Now we simplify, combine and solve as follows:
$\frac{-10t}{6t+2} = \frac{-5t^2+4}{3t^2+2t} \Rightarrow (-10t)(3t^2 + 2t) = (-5t^2 + 4)(6t + 2) \Rightarrow -30t^3 - 20t^2 = -30t^3 - 10t^2 + 24t + 8.$
Thus, $0 = 10t^2 + 24t + 8$ has solutions $t = \frac{-24 \pm \sqrt{24^2 - 4(10)(8)}}{20}$, which gives $t = -2$ or $t = -0.4$. The corresponding slopes are given by $\frac{dy}{dx} = \frac{-10t}{6t+2}$. At $t = -2$, you get $dy/dx = \frac{20}{-10} = -2$ and at $t = -0.4$, you get $dy/dx = \frac{4}{-0.4} = -10$.
Therefore, the two tangent lines are $y = -2(x - 0) + (-4)$ and $y = -10(x - 0) + (-4)$. Here is a picture: